INDIAN STATISTICAL INSTITUTE
BANGALORE CENTRE
B. MATH III YEAR, II SEMESTER(2003-2004)
SEMESTRAL EXAMINATION
ANALYSIS IV

Max. Marks: 100 Duration: 3hrs

1. Assuming that a complete metric space cannot be written as a countable union of nowhere dense sets, show that the intersection of a sequence of dense open sets in such a space in dense. [15]

2. Prove that the set Q of rational numbers is not a G_δ in R. [A G_δ is a countable intersection of open sets] [10]

3. Prove that any right continuous function $f : R \to R$ is Borel measurable. [15]

4. Lat $\{f_n\}$ be a sequence of real measurable functions on a measure space $(\Omega, \mathcal{F}, \mu)$ such that $\sup_n \int e^{f_n} d\mu < \infty$ and $f_n \to f$ *a.e.* If $\mu(\Omega) < \infty$ show that $\int |f_n - f| \, d\mu \to 0$. [20]

5.

a) Give an example of increasing sequence of measurable functions $\{f_n\}$ converging *a.e.* to f such that
$\int \lim f_n d\mu \neq \lim \int f_n d\mu$.

b) Give an example of a sequence of measurable functions $\{f_n\}$ such that $\int |f_n| \, d\mu < \infty$ for all n but
$\int \liminf f_n d\mu > \liminf \int f_n d\mu$. [5+5]

6. State and prove Jensen's inequality. [10]

7. If μ and ν are finite positive measures on $(\Omega, \mathcal{F})$ such that $\max(\mu(E), \nu(E))$ is a measure then show that either $\mu(E) \leq \nu(E)$ for all $E \in \mathcal{F}$ or $v(E) \leq \mu(E)$ for all $E \in \mathcal{F}$. [20]

8. Let f be a measurable function on a product space such that for each x, $f(x, y) = 0$ for almost all y. Show that there exists a null set A such that if $y \notin A$ then $f(x, y) = 0$ for almost all x. [10]